(This is a part of the book The Concept of Reality.pdf)

# 6. Reality and Mathematics

With the conclusion of the chapter on qualia, the project of this book is completed: to develop a concept of reality, which permits to treat "what is" and "why" questions not only formally but to follow them up to their metaphysical ground and from there to give the correct answers, and which makes it possible to think reality closed in itself and complete, including all phenomena that belong to it.

However, there is still something to be done: in 1.4, I have announced an explanation of why mathematics and logic do not exist "outside" the universe, but emerge from this universe and are part of it. This is the subject of this chapter.

## *6.1. Introduction: The Connection between Reality and Mathematics*

What is mathematics? The science of relations between objects and the structures evolving thereof.

What is reality *for us?* Relations between objects and the structures evolving thereof.

With this, most directly a close connection between mathematics and reality becomes apparent, which however only at first glance appears as possible identity of reality and a corresponding mathematical structure, because already the next intellectual step proves – as elucidated at the beginning of the Second Part – that there is an insurmountable difference between reality and its descriptions:

The objects of reality *exist,* whereas the objects of descriptions are only *defined.* Therefore mathematics lacks – as well as any other description system – the *substance* and, accordingly, the metaphysical quality *activity.*

However this does not mean that the just determined close connection between mathematics and reality is canceled. It only means that reality can never be identical with a mathematical structure, and that the necessary remaining difference is of a metaphysical kind.

But even if this difference is a metaphysical one, it must still manifest itself also formally: if it remained formally invisible, then *for us* the difference between mathematics and reality would be without consequence, and then it would not make sense to claim such a difference.

How the difference between mathematics and reality manifests itself as limitation of the formal describability of real systems has been one of the subjects of the previous chapters. Here is a short summary:

Every mathematical system consists of *a given set of axioms and rules.*

By contrast, the reality – due to its metaphysical quality *activity* – permanently produces *new rules* that cannot be derived from the given ones. So if one tries to map the reality onto a mathematical system, then the reality will incessantly produce states, which – expressed as propositions of the system – correspond to Gödel propositions, i.e. to non-derivable propositions.

This means: *the reality transcends every possible mathematical system.*

However, we know that many areas of the reality can be described by a mathematical system with great accuracy. Can we assume that we will – ultimately – find the best mathematical approximation? Or is it possible that the description of reality requires a mathematical structure which – given from where we have to start – is not accessible to us?

This seems unlikely. The kind of the connection between mathematics and physics on the one hand and reality on the other hand speaks against it: Mathematics begins with counting, and physics begins with measuring. Reality is countable and measurable, and in many cases the measuring values exhibit regularities. Why this is the case, has been elucidated in the initial chapter of the Third Part: Reality *emerges* from a law, it *is* the law – insofar the term *law* is understood not only formally but metaphysically, i.e. as *that which executes itself.* (On the question of countability, I'll be right back.)

In summary, it can be stated:

On the one hand, it is by no means mysterious or surprising why mathematics is suitable for the description of reality, but evident. On the other hand, there are also limitations, which, in the view of the reality presented here, ultimately go back to the fact that it is impossible to imitate through mathematical methods how reality generates the future, because it is performed in a non-algorithmic way, which presupposes the metaphysical quality *activity.*

Nonetheless, among all possible descriptions, mathematics is the one that represents the accidents of the objects of reality – i.e. the *structure* of the objective reality – most accurately, as long as the described entities and processes belong to the realm of *matter*.

Also the *origin of everything,* which *in itself* does not divide into substance and accident, can, as that what it is *for us* – as ever changing continuum –, be captured mathematically through the concept of the mathematical limit.

For an understanding of *mental phenomena*, however, a description which is adapted to the essence of the mind – the unity of SENSATION and meaning – is far more suitable. Here, mathematics and natural science have only assistive function; the attempt to capture mind in a scientific way reveals immediately the metaphysical shortcomings of mathematical and scientific concepts and methods.

## *6.2. Short Excursus: the three Worlds*

I consider the division of the realm of the existing into the world of material objects (world 1), the world of the mind (world 2) and the world of intellectual products (world 3) right and necessary. [1]

As long as it is a mere enumeration, however, it is unsatisfactory. [2]

With the concepts developed here, it is easy to transform this list into a structural hypothesis about reality by answering the following questions:

- Which reasons compel us to divide the world in this way?
- How are the three worlds interconnected? How can worlds 2 and 3 act on world 1?
- How do the objects of different worlds differ from each other?

All three questions have already been answered here, however without having been mentioned explicitly.

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[1] Short and concise in: *Karl R. Popper:* Three Worlds: The Tanner Lecture on Human Values at the University of Michigan, April 7th, 1978

[2] Popper's arguments for this classification are not compelling, because all his statements can easily be transformed into reductionistic statements. They simply lack a *systematic* justification.

In our terminology, world 1 is the world of the *first substance*, world 2 is the world of the *second substance*, and world 3 is the world *without substance*.

Is this not just another enumeration? Not at all! As a reminder, here is a quick recapitulation of the thought trains that contain the required answers:

The introduction of the concept *substance* is necessary in order to do justice to the obvious fact that objects of reality – as opposed to objects in a description – are *active*, and that this activity must come *from something* that does not appear as such in the descriptions.

This can be grasped conceptually through the statement that objects of reality consist of *substance and accidents*, whereas objects of descriptions consist *only of accidents*. They are substance-less.

In objects of the reality, substance and accidents are inextricably linked with one another. This follows immediately from the fact that it is impossible to deprive real objects of their *activity* – obviously it is impossible to separate the earth from its gravity.

Nature unfolds to ever more complex structures. As long as the more complex accidents can be described as functions of simpler accidents, these accidents remain connected through a chain of functional dependencies with the simplest accident – the first accident – and therefore also with the according substance – the first substance.

But the accidents of mental entities – intrinsic meanings – cannot be regarded as functions of simpler accidents. Therefore, their connection with the initially introduced substance is interrupted, and this enforces the introduction of another substance or of a transformed substance, with which the accident *intrinsic meaning* is inextricably linked.

This second substance must be exactly that, what descriptions of mental entities are lacking, and that is *sensation.* Through the emergence of mental beings, the first substance AGENT or SPACETIME transforms into the second substance SENSATION.

As can be seen, all these statements are direct consequences of the insight, which stood at the beginning: the insight that objects of reality are *active,* and that this activity must emanate *from something* or belong *to something;* thus, everything follows from the assumption of the *substance.*

The definitions of the three worlds, however, are already contained in these statements.

This means: The classification of the world into a material world, a mental world and a world of the products of the mind is a direct consequence of the (necessary) introduction of the substance. The connections between the three worlds as well as their differences are explained by the above conclusions in a simple manner.

Further important points are the following ones:

In order to justify the introduction of worlds 2 and 3, it is necessary to refute the hypothesis that mind can be reduced to matter. The according argumentation has been carried out in chapters *3. Free Will* and *4. The modified Picture of Reality.*

Moreover, the assertion that world 2 and world 3 can act causally on world 1, presupposes the concept of *causality top-down.* This concept has been introduced through examples and analogies in chapter 3 and justified systematically in chapter 4.

With this, everything of importance about the introduction of the three worlds and their interconnection has been said.

Regarding world 3 – the world of the products of the mind –, however, hitherto only some examples have been discussed. (One of these examples was *descriptions of reality,* another one, which has been described in more detail, was *simulations of actually existing systems.*)

Therefore now, before we come (in the next section) to the actual topic of this chapter – the question of the kind of existence of mathematical objects and propositions –, some general remarks on the entities of world 3 shall follow.

The main features of the entities in world 3 are exactly those which have been named in the section on simulation: The entities in world 3 differ from the entities of the other two worlds in that they have no substance and consist only of accidents. Therefore they cannot exist independently; they require a *material basis*.

These accidents can be mental or material. Examples of the first kind are books, compositions, or sanctuaries, examples of the second kind are apparatus, cars, or rockets.

Why have the entities in world 3 no substance? Because they lack *activity* or *sensation.* If they are constructed systems with their own dynamics, like a tellurion or any other technical device, they must be provided with *activity* from outside; if they *mean* something, like artworks or temples, then they

enter the connection with the according substance SENSATION only if they are perceived and understood by an entity that possesses mind.

A necessary condition for a world 3 object is that it owes its existence to an *intention.*

Is this condition also sufficient? What about objects that have no dynamics, but merely fulfill a purpose, such as tables? Do they too belong to world 3?

This is a question of definition. But since the introduction of world 3 is inevitable and its existence is thus secured, it seems reasonable to class all objects that owe their existence to an intention under the objects of world 3.

## *6.3. What Kind of Existence have mathematical Objects and Theorems?*

Mathematics begins with counting.

The fact that the numbers, with which one counts, are called natural numbers, however, is misleading, because they are certainly not *natural*. There are no numbers in nature, or, more exactly: in world 1.

*We* are the ones who count, in other words, numbers belong to descriptions of the material reality and not to the material reality itself.

If one considers the numbers with which to count as natural, then, after a series of steps that appear evident, one is faced with the imaginary unit, seeking in vain for its ontological status.

If, however, it is clear from the outset that there are no numbers in nature, then the difference between natural and complex numbers is only that the relationship of the natural numbers to elements of the physical reality is simpler than that of the complex numbers.

Numbers can appear in world 2 and in world 3:

- They can be elements of mental processes, in which they appear as mental entities or as accidents of mental entities.

- If they are elements of descriptions, which are attached in some way – e.g. printed – to a material basis using any kind of code, then they are entities of world 3.

World 1 does not contain numbers, but it is countable. Why is it countable?

The first prerequisite lies in us ourselves. As was shown in *3.4. Organized States in Neuronal Networks,* we can neither think nor perceive the Individual, but only the General.[3]

Natural numbers are such Generals, through which we perceive and think reality.

The second condition is that nature generates *objects* – stationary states of the continuum dynamics – which are sufficiently similar to each other to enable the formation of neuronal attractors that represent them, or, in other words, which fall under the same concept. Since at first the universe organizes itself *globally,* such that everywhere similar conditions for the local self-organization develop, this assumption seems plausible. Moreover, we can simply *see* that it applies.

Initially, the development of mathematics follows the path that is determined by the natural numbers and the well-known arithmetic operations. Thereafter, however, it turns into a free play of the mind with objects and structures, which is a characteristic of world 2.

Mathematics is the most outstanding example of a special kind of mental activity: the invention and elaboration of systems, which consist *firstly* of a number of defined objects, and *secondly* of rules how additional objects can be constructed from existing objects. These systems are then at the same time objects of world 3.

An example of such an entity is the carpet which was mentioned in section *4.3. Why Nature is not an algorithmic System.* There stood the following: (I repeat the whole passage, because it illustrates the essential point of the question what kind of existence mathematical objects and theorems have.)

"Suppose we have the intention to weave a multicolored carpet. The initial series of meshes lies already before us, and we also have a complete set of weaving-rules. Let us now assume that, at some point during the course of the weaving process, on the carpet the image of a lion arises. The question is: did this lion already exist before the carpet was woven? If this means that the lion can be produced by the initial series of meshes and the weaving-rules – that, in this sense, it is thus contained in them – then the answer is *yes.*

[3] As a reminder: Patterns, which represent something, are attractors in the state space of the network. Attractors have a basin. Points in the basin correspond to stimuli that are triggered by an individual case, and the basin as a whole corresponds to all possible individual cases, which lead to the same attractor. The attractor itself is therefore the representation of the general over these individual cases. Thus, understood as a mental entity, it is not an Individual but a *Universal.*

Mathematicians are confronted with a question of the same kind, when they encounter mathematical theorems during the course of their conclusions. These theorems are obviously not invented but discovered. They are in the same way "contained" in the axioms and rules of the mathematical system, as the lion is contained in the initial series of meshes and the weaving-rules of the carpet system."

Therefore it is clear which kind of existence mathematical objects and theorems have: they are elements of systems, which are devised in world 2, i.e. by entities that possess mind.

If in the system a procedure exists, through which one can, after a finite number of steps, arrive at such an element, then it can be *discovered*, and therefore it seems justified to claim that this element has existed already before its discovery.

Thus, mathematical objects and theorems exist if and only if a system exists, according to whose axioms and rules they are formed. Through application of the rules for constructing new objects and propositions they can be discovered.

So the following can be asserted:

Mathematical entities are created in world 2 and exist then in world 3. Therefore, mind is a necessary condition for the existence of mathematical entities – as it is for all entities of world 3. In short: without mind no mathematics.

Some mathematicians and philosophers believe, however, that mathematical objects and statements have a Platonic existence, i.e. that they exist entirely independently, in the form of an autonomous reality. The reason for this believe is that mathematical propositions seem to be true regardless of their material realization. The ratio of circumference and diameter of a circle will always be $\pi$ – and one is tempted to say, no matter what universe one is in or even whether a universe exists.

So why is that the case? To investigate this issue, we first focus on the question whether numbers and basic arithmetic operations are *invented* or *discovered*.

The answer follows from the just mentioned fact that the mind contains not individuals but only generals. Any object that appears in our perception or in our thinking is a *universal.* It can only be understood as individual by assigning to it a name, or a position and a point in time, or through a sufficient number of characteristics, which however, considered alone for themselves, are again universals.

From this follows that, for entities that possess mind, the world is divided into *sets.*

Numbers, however, are nothing other than properties of such sets. (E.g. the number 5 is that property which is common to all sets that contain exactly as many elements as I have fingers on one hand.) In other words: counting is a fundamental act of any sufficiently developed mind; therefore it appears *with necessity* in such a mind.

The next step: The elementary operations with numbers originate from experiences made through the handling of objects: 2 sheep plus 1 sheep equals 3 sheep, and this is a law that applies regardless of whether there are sheep, and even regardless of what is being counted, i.e. regardless of its material realization.

Does this law possess a platonic existence? No. It can only occur if the natural evolution has produced beings with mind, who understand the world conceptually and divide it therefore into sets of objects.

Did this law exist, *before* it appeared in the mind? No. Nothing exists before it emerges. Also mind does not exist, before it evolves, and this assertion would hold true even if it could be demonstrated that the evolution of mind *is necessary,* in the sense that every possible evolution of the cosmos must bring forth mind. Also in this case, it would not be reasonable to claim that the mind existed before it actually appeared.

The same applies to mathematical objects and theorems. They appear as necessary consequence of the relationship between mind and material reality. Only with the appearance of mind they can exist; before that, they have no existence.

Thus, the question of whether numbers and arithmetic operations are invented or discovered, can only be answered with "neither-nor".

Are they *discovered*? No. They did not exist before. Are they *invented*? No. The development of mind led to their appearance *with necessity.*

So even if one assumes that *any* being that has mind and is to a sufficient degree capable of thinking, must arrive at the numbers and at mathematics, this does not prove their independent existence. Rather it is the *interaction* of world 2 and world 1, which necessarily leads to mathematics, and not the platonic existence of mathematical concepts and propositions.

Numbers are elements of the worlds 2 and 3. In world 1, the material world, there are no numbers. Neither are there circles, circumferences or diameters. Also the lion, that appears on the carpet, is not a materially existing lion, however the assertion that it appears *if* the carpet is woven is true, regardless whether it is actually woven or not.

*None* of the objects of world 3 is identical with an object of world 1. This applies also in those cases, where the temporal development of an actually existing system can approximately be represented by a mathematical system. Also the objects of such a mathematical system have no material existence. The mathematical system *is not* the real system, and the objects of the mathematical system *are not* the real objects.

From this *metaphysical* difference follows a *formal* difference. Objects, which exist in a mathematical system, are generated by the axioms and rules of the system in an algorithmic way. Actually existing objects, however, evolve from the fundamental law and the global conditions in a non-algorithmic way.

As stated before: during its unfolding through self-organization, reality sometimes approaches algorithmic describability – then mathematics and reality seem to touch each other –, but without ever completely reaching it. And at times real structures or paths of real objects resemble mathematical figures, like circles or ellipses, without ever entirely conform to them, and actually existing systems occasionally resemble physical systems that obey a law.

But ultimately, all natural laws – except the *fundamental* law, which however remains limited to the infinite Small – are about idealized systems, which are never completely realized. Therefore, natural laws, not other than circles or ellipses, are elements of devised worlds, which are only simulating the real world and which, for metaphysical reasons, can never fully conform with it.

## *6.4. The Source of the General*

How does the General come into the world? In two ways:

1. Reality is generated from a single rule, which is fact and law at the same time.

The universe develops from this differential rule through self-organization. First, a global spacetime pattern is formed whose individual areas represent boundary conditions for the emergence of local spacetime patterns. These local patterns ("elementary particles") are sufficiently similar to each other so as to act as objects of natural laws. They in turn form patterns of higher order to which the same applies. This process is repeated a few times.

The objects, which develop in this way, present themselves to us as that what is the case, as the respective *Individual*. As individuals, however, they could not behave according to laws – for that the *General* is required which they are made from and which they carry in themselves.

So this is the first way in which the General enters reality: The rule which reality is made from is itself a General, and its general validity is inherited to the laws of the relationships between the spacetime patterns (objects) emerging from it – but only *approximately*, since there are only objects and circumstances that are *similar* to each other, and none that are *identical*; For the exact validity of such laws, however, identity of the objects and circumstances would be required.

In order to be able to formulate this fact at all, however, the General must already be present in the mind. It comes into the mind in the following way:

2. In the mind, all objects and all facts are represented by *attractors* of the dynamics of the system (the neural network) which generates the mind.

Attractors, however, cannot represent individual objects but only sets of (similar) objects: from the fact that each attractor has a basin of attraction follows that *similar objects* are represented by the *same attractor*. Thus, every sensation, every concept is a *Universal*.

Since all (mental) objects and facts are Universals, the mental reality contains exclusively statements about Universals. Therefore, in the realm of what is perceived and thought, laws can be perfectly accurate, and true statements are possible. Applied to the material reality, however, they remain true only as long as the actually existing differences are disregarded – as e.g. when counting objects or performing calculations with objects. But in the description of the dynamics of real systems, it is neither possible to disregard the existing differences, nor can they be fully covered. So there are only approximations *on principle*.

To understand the relationship between natural laws and reality, both sources of the General are needed:

Without the knowledge of the fundamental law that creates reality, it cannot be understood why reality behaves according to laws and repeatedly approaches algorithmic describability; Hume's problem of the justification of laws is then unsolvable.

Without the knowledge that objects and facts are represented by attractors, the origin of the General remains hidden.

If the knowledge of one of the two sources was missing, then the adoption of a separate, Platonic existence of the General would be inevitable. However, like any kind of dualism or pluralism, this assumption would fail due to the unsolvability of the problem of the interaction: The question of how laws and objects are interrelated – that is: how the General *acts on* the individual being, or how it *is in* this being – could not be answered.

## *6.5. The Origin of Reality and of Mathematics*

Everywhere and anytime the fundamental law is in effect and, by its action, generates the ever-changing fabric of spacetime. Simple objects emerge in the form of spacetime patterns. They join together to form objects of higher complexity. This process is repeated several times. Finally, objects evolve that are capable of replicating themselves. With this, the biological evolution starts. At last, it leads to objects that are equipped with mind. These objects – or let us better call them *beings* – capture the world through concepts which are universals. So they are led to the world of numbers. There, they discover the infinitely Small and grasp it through the concept of the mathematical limit. Equipped with this knowledge, they turn to the *origin of everything* and understand how reality unfolds by changing from instant to instant according to the fundamental law.

So this is the short version of the story of the emergence of mathematics and how it arrives at the *origin of everything*.

Reality itself does not apply mathematics. Just as the blade of grass does not calculate where to move but simply follows the wind that touches it, also the reality does not calculate its next step, but simply follows everywhere and anytime the differentially adjacent spacetime changes.

We, however, need mathematics to understand and describe how the fabric of reality unfolds, because, due to the lack of the *substance* and, with it, of the metaphysical quality *activity,* our descriptions are inappropriate to imitate directly what reality does.